Birzeit University
Department of Mathematics
math 243

Final exam . Spring 2021
Name : …………….. Number:………

Question #1((20 %) Prove or disprove each of the following statements

a) For any sets A, B we have $\rho(A-B)=\rho(A)-\rho(B)$.

b) $(\forall x\in\mathbb{N})(\exists y\in\mathbb{N})(x>y)$ where $\mathbb{N}=\{1,2,3,4,\ldots\}$

c) $\chi_{A\cap B}=\chi_A\cdot\chi_B$

d) $\rho(A\cup B)=\rho(A)\cup\rho(B)$

Question #**2**(12% $\in \mathbb{R}^{*}$ then $x^{2} - 2x + 5$ is greater than or equal to 4.

2)Let A,B,C be sets , Prove that $A-(B \cap C)=(A-B) \cup (A-C)$

Question #3((12%) Which of the following statements is true and which is false

1)......... $\{1,\{2\}\} \in \{1,\{2\},\{1,2\}\}$

2)......... $\phi \subseteq \{1,\{\phi\}\}$

3)......... $P \to \sim P$ is a contradiction

4)......... Any reflexive and symmetric relation on A is transitive

5)......... for any sets A, B it is true that $A = (A - B) \cup (A \cap B)$

6)......... If $x \notin A$ and $A \notin B$ then $x \notin B$ where A,B are sets

7)......... Any reflexive relation on a set X is transitive

8)......... There is no onto function from A={1,2,3} onto B={a,b,c,d}

9)......... If A={1,2,3} then the number of functions from A to A is 6

10)......... $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R})(z^2 = xy)$

11)........... All students in math 243 this semester enjoy this course

12) .... If R and S are transive then $R \cup S$ is transitive

Question #4((12%) Let $f : \mathbb{R} \to \mathbb{R}$ be the ~~greatest integer~~ function

$x \to [x]+2$ where $[x]$ is the greatest integer of x.

Let A=[1,5] , B =(0,5] .Find

a) f(A)=

b) $f^{-1}(B)$=

c) $f^{-1}(f(A))$=

d) $f(f^{-1}(B))$ =

Question #5((10%) Let $f: X \rightarrow Y$ be a function .Prove that f is one to one if and only if $f(A \cap B)=f(A) \cap f(B)$ for all A,B subsets of X.

Question #6((10%) Use mathematical induction to prove that $\cancel{\forall n \in \mathbb{N}, n \geq 4, n^2 < 3^n}$"

$$\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \overline{A_i}$$

Question#7 (14%)

Let f(x) $\sqrt{2x-5}$,g(x)=$\sqrt{18-x}$

a) find domain of f and g

b) write explicit expression for f ∘ g ang g ∘ f

c) find domain of f ∘ g ang g ∘ f

d)write explicit expression for $(f \circ g)^{-1}$ ang $(g \circ f)^{-1}$

Question #8((10%) Let f, g be functions with domains A and B respectively and let $C = A \cap B$ prove that $f \cup g$ is a function if and only if $f_{|C} = g_{|C}$